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ON THE STABILITY OF THE VERTICAL ROTATION OF A SOLID SUSPENDED ON A ROD*

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The problem of the motion of a dynamically symmetric solid suspended from a fixed point by a weightless rod and two ball and socket joints one of which is fixed at the fixed point O', and the other is on the body axis of symmetry at the point O is considered. The question of the stability of the uniform body rotation when points O' and O, and the body centre of inertia C lie on the same vertical, and at the same time point O may be either above or below point O', and point C either above or below point O, is discussed. An analysis of the necessary and sufficient conditions for stability is given. The set of all the system's parameters is reduced to three independent dimensionless parameters L, Ω and β, and in the plane (L, Ω), for fixed values of β, the regions for which the unperturbed rotation is steady, or steady to a first approximation, or non-steady are indicated. The regions for which the body rotation is steady to a first approximation when the point O is situated higher than the point O', and the point C lies higher or lower than the point O are determined.

The sufficient conditions for the vertical rotation of a dynamically symmetric body suspended on a filament were obtained in /1/ and investigated for the cases where in non-perturbed motion the point C is below point O, when points C and O coincide, and when the length of the filament is zero (Lagrange gyroscope). In /2/ an analysis is given of the sufficient conditions for stability obtained in /1/, and also the necessary conditions for the cases where in a non-perturbed motion point C is located above point O.

1. Consider, in a uniform field of gravity, the motion of a dynamically symmetric solid suspended on a thin straight weightless rod and two ball and socket joints, one of them being the fixed point O', and the other located on the axis of symmetry of the body at point O.

We adopt the coordinate system $Ox_1x_2x_3$ whose axes are invariably linked with the body and directed along its principal axes of inertia for the point O. Let us introduce the following notation: m, $\mathbf{J}_C$ is the mass and the tensor of inertia of the body for its centre of mass C, with the diagonal elements $J_1 = J_2$, J_3; $\boldsymbol{\omega}$, $\mathbf{K}_C = J_C \cdot \boldsymbol{\omega}$, are the angular velocity and the momentum of the body, computed for point C, $\mathbf{a}$ is the radius vector of point C relatively to point O, $\mathbf{v}$ is the velocity of point O, $\boldsymbol{\gamma}$ is the unit vector of the upward vertical, l is the length of the rod, $\mathbf{e}$ is the unit vector directed along the rod to point O', g is the acceleration due to gravity, and N is the reaction of the rod. We shall express all vectors by their projections ω_i, $K_{Ci} = J_i\omega_i$, v_i, γ_i, e_i, a_i on the x_i axis $(i = 1, 2, 3)$ with $a_1 = a_2 = 0$, $a_3 = a$.

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The equation of motion in the coordinate system $Ox_1x_2x_3$ can be expressed in the form

$$m\,[d/dt\,(\mathbf{v} + \boldsymbol{\omega} \times \mathbf{a}) + \boldsymbol{\omega} \times (\mathbf{v} + \boldsymbol{\omega} \times \mathbf{a})] = -mg\boldsymbol{\gamma} + N\mathbf{e} \tag{1.1}$$

$$d\mathbf{K}_C/dt + \boldsymbol{\omega} \times \mathbf{K}_C = N\mathbf{e} \times \mathbf{a}$$

$$d\boldsymbol{\gamma}/dt + \boldsymbol{\omega} \times \boldsymbol{\gamma} = 0, \quad l\,(d\mathbf{e}/dt + \boldsymbol{\omega} \times \mathbf{e}) = -\mathbf{v}$$

The meaning of Eqs.(1.1) is obvious. They admit of the following first ingegrals

$$V_1 = \boldsymbol{\omega}\cdot\mathbf{J}_C\cdot\boldsymbol{\omega} + m\,(\mathbf{v} + \boldsymbol{\omega} \times \mathbf{a})^2 - 2mg\,(l\mathbf{e} - \mathbf{a})\cdot\boldsymbol{\gamma} = \text{const}$$

$$V_2 = [\mathbf{J}_C\cdot\boldsymbol{\omega} + m\,(\mathbf{a} - l\mathbf{e}) \times (\mathbf{v} + \boldsymbol{\omega} \times \mathbf{a})]\cdot\boldsymbol{\gamma} = \text{const}$$

$$V_3 = \omega_3 = \text{const}, \quad V_4 = \boldsymbol{\gamma}^2 = 1, \quad V_5 = \mathbf{e}^2 = 1, \quad V_6 = \mathbf{v}\cdot\mathbf{e} = 0$$

and have the partial solutions

$$v_i = \omega_i = \gamma_i = e_i\ (i = 1, 2), \quad v_3 = 0, \quad \omega_3 = \omega > 0,\ \gamma_3 = 1, \tag{1.2}$$

$$e_3 = \pm 1\ (N = \pm\, mg)$$

which describe the uniform rotations of the body for which the points O', O and C lie on the same vertical, where, for the first solution, $(e_3 = 1,\ N = mg)$ the point O is situated below point O', and for the second $(e_3 = -1,\ N = -mg)$ above this point. The solutions (1.2) can be regarded as one solution with $e_3 = 1$, if for the second solution we assume $l < 0$.

2. We retain in the perturbed motion the above notation of the variables. Then Eqs.(1.1), linearized in the vicinity of solution (1.2), take the form

$$le_*^{\cdot\cdot} + 2i\omega le_*^{\cdot} + (g - \omega^2 l)\,e_* - i\,(l - a)\,\omega_*^{\cdot} - \omega\,(l - a)\,\omega_* - g\gamma_* = 0 \tag{2.1}$$

$$J_1\omega_*^{\cdot} + i\omega\,(J_1 - J_3)\,\omega_* + imgae_* = 0, \quad \gamma_*^{\cdot} + i\omega\gamma_* - i\omega_* = 0$$

$$e_* = e_1 + ie_2, \quad \omega_* = \omega_1 + i\omega_2, \quad \gamma_* = \gamma_1 + i\gamma_2$$

where $e_3 = 1,\ \gamma_3 = 0,\ v_3 = 1,\ N = mg\,\mathrm{sign}\,l$ to a first approximation. Finding the solutions of Eqs.(2.1)in the form

$$(\omega_*, \gamma_*, e_*) = (\omega_*^{\circ}, \gamma_*^{\circ}, e_*^{\circ})\exp\,[i\,(\lambda - \omega)\,t]$$

we obtain the following characteristic equations for determining the constant λ :

$$g\Delta_0(\lambda) - \lambda^2 l\Delta_1(\lambda) = 0, \quad \Delta_0 = -J_1^*\lambda^2 + J_3\omega\lambda - mga \tag{2.2}$$

$$\Delta_1 = -J_1\lambda^2 + J_3\omega\lambda - mga, \quad J_1^* = J_1 + ma^2$$

Thus, for the motion (1.2) to be steady with respect to the variables $\omega_i,\ \gamma_i,\ e_i\ (i = 1, 2, 3)$ it is necessary that all four roots of Eq.(2.2) be real.

3. We obtain the sufficient conditions for stability of the solution (1.2) from the Routh theory as the conditions for the following group of integrals to be positive definite (λ is a parameter):

$$V = V_1 - 2\lambda V_2 - 2J_3\,(\omega - \lambda)\,V_3 + [J_3\omega\lambda + mg\,(l - a)\,]V_4 + mgl\,V_5$$

on a linear manifold determined by the equations $\delta V_i = 0\ (i = 2, \ldots, 6)$, i.e. for $\delta\omega_3 = \delta\gamma_3 = \delta e_3 = v_3 = 0$. Introducing the new variables $\Omega_j,\ \alpha_j\ (j = 1, 2)$: $\omega_j = \Omega_j + \lambda\gamma_j,\ e_j = \gamma_j + \alpha_j$ instead of ω_j and e_j we represent V as

$$V = J_1^*(\Omega_1^2 + \Omega_2^2) + \Delta_0(\lambda)(\gamma_1^2 + \gamma_2^2) + mgl\,(\alpha_1^2 + \alpha_2^2) + m\,(v_1^2 + v_2^2) - 2\,mal\,\lambda\,[(\Omega_1 + \lambda\gamma_1)\,\alpha_1 + (\Omega_2 + \lambda\gamma_2)\alpha_2] + 2ml\lambda\,(\alpha_1 v_2 + \alpha_2 v_1) - 2a\,(\Omega_1 v_2 + \Omega_2 v_1) + \ldots$$

where terms of the second order of smallness are ignored. The conditions for the function V to be positive definite are reduced to the inequalities

$$\Delta_0(\lambda) > 0, \quad l\,[g\Delta_0(\lambda) - \lambda^2 l\Delta_1(\lambda)] > 0 \tag{3.1}$$

When these conditions are satisfied, the function V is the first integral), with its sign fixed with respect to $\omega_i,\ \gamma_i,\ e_i,\ v_i\ (i = 1, 2, 3)$, of the equations of perturbed motion. Hence, by the Lyapunov stability theorem we conclude that the inequalities (3.1) are the sufficient conditions of stability of solution (1.2) with respect to the variables $\omega_i,\ \gamma_i,\ e_i,\ v_i\ (i = 1, 2, 3)$.

Thus, the sufficient conditions for stability of the unperturbed motion (1.2) are such that for a certain real value of the parameter λ, both inequalities (3.1) are satisfied simultaneously.

In analysing the roots of Eq.(2.2) and the inequalities (3.1), let us agree to assume that $a > 0\ (a < 0)$ if for the solution (1.2) point C is above (below) point O.

Conditions (3.1) were obtained in /1/ and were discussed for the following cases: 1) $l = 0$; 2) $l > 0,\ a < 0$; 3) $l > 0,\ a = 0$.

4. Conditions (3.1) cannot be satisfied for $l < 0$ because then the inequality

$l\,[g\Delta_0(\lambda) - \lambda^2 l \Delta_1(\lambda)] < 0$ would be valid if $\Delta_0(\lambda) > 0$. Therefore, we shall assume below that $l > 0$.

To analyse conditions (3.1) we shall introduce the function

$$l(\lambda) = g\Delta_0(\lambda)/(\lambda^2\Delta_1(\lambda)) \tag{4.1}$$

The graph of this function is shown in Fig.1 for the case where

$$J_3^2\omega^2 - 4J_1^* mga > 0 \tag{4.2}$$

The letters λ_1, λ_2 and $\lambda_1^\circ, \lambda_2^\circ$ denote the roots of the equations $\Delta_0(\lambda) = 0$ and $\Delta_1(\lambda) = 0$ respectively, and l_* and l^* $(l_* < l^*)$ are the experimental values of the function (4.1).

To satisfy the first condition (3.1) it is necessary for inequality (4.2) to be satisfied and then $\Delta_0(\lambda) > 0$, if $\lambda_1 < \lambda < \lambda_2$. The second equation is equivalent to the condition $l > l(\lambda)$, if $\lambda < \lambda_1^\circ$ $(\lambda \neq 0)$ or $\lambda > \lambda_2^\circ$, and to $0 < l < l(\lambda)$, if $\lambda_1 < \lambda < \lambda_2$. Therefore conditions (3.1) are equivalent to the conditions

$$\Delta_0(\lambda) > 0,\ l > l(\lambda),\ \text{if}\ \lambda < \lambda_1^\circ\ (\lambda \neq 0)\ \text{or}\ \lambda > \lambda_2^\circ \tag{4.3}$$

$$\Delta_0(\lambda) > 0,\ 0 < l < l(\lambda),\ \text{if}\ \lambda_1 < \lambda < \lambda_2 \tag{4.4}$$

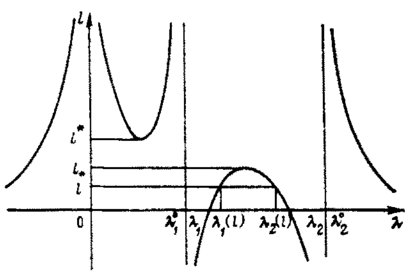

Fig.1

Conditions (4.3) cannot be satisfied for any of the values of λ: $\lambda < \lambda_1^\circ$ $(\lambda \neq 0)$, $\lambda > \lambda_2^\circ$ because $\Delta_0(\lambda) < 0$ for all $\lambda < \lambda_1^\circ$, $\lambda > \lambda_2^\circ$.

To analyse conditions (4.4) we take an arbitrary value of l, $0 < l < l_*$, and denote by $\lambda_1(l), \lambda_2(l)$ the roots of the equation $l(\lambda) = l$ which satisfy the inequalities $\lambda_1 < \lambda_1(l) < \lambda_2(l) < \lambda_2$ (Fig.1). Then conditions (4.4) will be satisfied for all values of λ which satisfy conditions $\lambda_1(l) < \lambda < \lambda_2(l)$. Further, we see from Fig.1 that the equation $l(\lambda) = l$ has four real roots if $l > l^*$ or $0 < l < l_*$, and two real and a pair of complex roots if $l_* < l < l^*$. When $l = l_*$ and $l = l^*$, this equation has equal real roots and a pair of complex roots. Since, for $l \neq 0$ the equation $l(\lambda) = l$ is identical to Eq.(2.2), it follows that the necessary stability conditions are satisfied for $l > l^*$ and two $0 < l < l_*$. Comparing this result with the analysis of conditions (4.3) and (4.4) discussed we finally conclude that: 1) for $l > l^*$ the necessary stability conditions are satisfied, but not the sufficient conditions (4.3); 2) for $0 < l < l_*$ both the necessary and sufficient conditions are satisfied simultaneously.

It is asserted in /1/ that for $l > 0$ and $a > 0$ conditions (3.1) are reduced to the requirement that all four roots of the equation $l(\lambda) = l$ should be real with respect to λ, and in /2/ it is additionally mentioned that for (1.2) the necessary stability conditions are identical with the sufficient conditions, and reduce to the requirement that all roots of Eq.(2.2) should be real. As the analysis above shows, these assertions only hold for $0 < l < l_*$, and do not apply to the case when $l > l^*$.

5. Let us substitute $\lambda = J_3 J_1^{-1}\omega x$ into (2.2), and represent these equations in the form

$$L\Omega^2(x^4 - x^3) + (L - \beta)\Omega x^2 - \Omega x - 1 = 0 \tag{5.1}$$
$$L = \frac{mal}{J_1},\quad \Omega = \frac{J_3^2\omega^2}{J_1 mga},\quad \beta = \frac{J_1^*}{J_1} = 1 + \varepsilon \geqslant 1$$

For all four roots of Eq.(5.1) to be real and different it is necessary and sufificent to satisfy the following conditions for $L > 0$

$$\begin{aligned}
&\Delta_3^1 = L^2\Omega^5\Delta_3 > 0,\quad \Delta_5^1 = L^2\Omega^7\Delta_5 > 0,\quad \Delta_7^1 = L^2\Omega^6\Delta_7 > 0 \\
&\Delta_3(L, \Omega, \beta) = 3\Omega L - 8(L - \beta) \\
&\Delta_5(L, \Omega, \beta) = 3\Omega^2 L^2 + [(L - \beta)^2 - 8(L - \beta) - \\
&\quad 6(\beta - 3)]\,\Omega L - 4(L - \beta)[(L - \beta)^2 + 4L] \\
&\Delta_7(L, \Omega, \varepsilon) = 4(\Omega - 4)[\Omega L - (L - 1)^2]^2 - \\
&\quad 4\varepsilon(L - 1)[5L\Omega^2 - (3L^2 - 10L + 3)\Omega - 16(L + 1)^2] + \\
&\quad \varepsilon^2[L\Omega^2 + 4(3L^2 - 26L - 3)\Omega - 32(3L^2 - 2L + 3)] - \\
&\quad 4\varepsilon^3(L - 1)(\Omega - 16) - 16\varepsilon^4
\end{aligned} \tag{5.2}$$

(see /3/, p.60).

If $L < 0$, the signs of all the inequalities in (5.2) should be changed.

If $\Delta_7^1 = 0$, Eq.(5.1) has multiple roots. For $L > 0$, it has two real and a pair of complex roots if $\Delta_7^1 < 0$, and has no real roots if $\Delta_7^1 > 0$ and neither of the first two inequalities in (5.2) is satisfied. For $L < 0$, Eq.(5.1) has two real and a pair of complex roots if $\Delta_7^1 > 0$, and has no real roots if $\Delta_7^1 < 0$ and, in addition, at least one of the inequalities $\Delta_3^1 > 0$, $\Delta_5^1 > 0$ is satisfied.

The analysis of conditions (5.2) reduces to constructing in the (L, Ω)-plane the curves defined by the equation $\Delta_3 = 0$, $\Delta_5 = 0$, $\Delta_7 = 0$ for a fixed values of the parameter $\beta = 1 + \varepsilon \geqslant 1$. This does not present any difficulty for the first two of these equations.

Constructing the curves determined by the equation $\Delta_7 = 0$ is more complex. For $\varepsilon = 0$, this equation takes the form

$$\Delta_7 (L, \Omega, 0) = 4 (\Omega - 4)[\Omega L - (L + 1)^2]^2 = 0$$

and breaks up into two equations, $\Omega = 4$ and $\Omega = (L + 1)^2 L^{-1}$ which define the curves shown in Fig.2 (the dashed lines).

Let us introduce new variables, $w = \Omega - 4$ and $Z = L - 1$, and write the equation $\Delta_7 = 0$ as

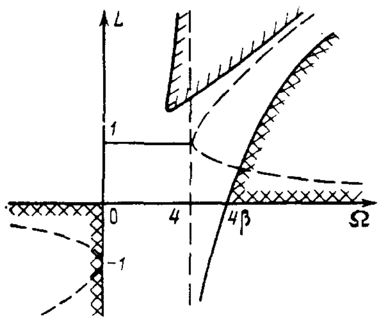

Fig.2

$$\Delta_7 = a_0 w^3 + a_1 w^2 + a_2 w + a_3 = 0 \tag{5.3}$$

$$a_0 = 4 (z + 1)^2,$$
$$a_1 = -(8z^2 + 20z\varepsilon - \varepsilon^2)(z + 1)$$
$$a_2 = 4[z (z - \varepsilon)^3 - 18\varepsilon \times (2z + \varepsilon)(z + 1)],$$
$$a_3 = 16\varepsilon [(z - \varepsilon)^3 - 27\varepsilon (z + 1)]$$

We denote by $D (z, \varepsilon)$ the discriminant of this equation

$$D (z, \varepsilon) = - 4p^3 - 27q^2 = \frac{\varepsilon Q (z, \varepsilon)}{16 (z + 1)^6} \tag{5.4}$$

$$3a_0^2 p = 3a_0 a_2 - a_1^2, \quad 27a_0^3 q = 27a_0^2 a_3 - 9a_0 a_1 a_2 + 2a_1^3$$

where $Q (z, \varepsilon)$ is the polynomial in each of the variables z and ε, which, with respect to z, is of degree 11, and with respect to ε of degree 9, in which case

$$Q (z, 0) = 512z^9 (z + 2)^2, \quad Q (-1, \varepsilon) = (\varepsilon + 1)^6(\varepsilon - 8)^3$$
$$Q (\varepsilon, \varepsilon) = -2^6 3^9 \varepsilon^3 (\varepsilon + 1)^3(\varepsilon + 2)^2$$

Hence it follows that for each ε the function Q, and therefore, by Eq. (5.4) the discriminant D, has as least one real root $z_1 = z_1(\varepsilon)$ if $0 < \varepsilon < 8$, and at least three real roots $z_3 (\varepsilon) < 0 < z_2 (\varepsilon) < z_1 (\varepsilon)$, if $\varepsilon > 8$; at the same time for $0 < \varepsilon < 8$ we have $D > 0$ if $z > z_1$, $D < 0$ if $z < z_1$, and for $\varepsilon > 8$ we obtain $D > 0$ if $z > z_1$ or $z_3 < z < z_2$, and $D < 0$ if $z_2 < z < z_1$ or $z < z_3$. For this reason Eq. (5.3) for $0 < \varepsilon < 8$ has three real roots if $z > z_1$, and one real root if $z < z_1$. For $\varepsilon > 8$, Eq. (5.3) has three real roots if $z > z_1$, or $z_3 < z < z_2$, and one real root if $z_2 < z < z_1$ or $z < z_3$.

Let us denote by $L = L (\Omega, \beta)$ a real algebraic function determined by the equation $\Delta_7 = 0$. The following expansions hold for the branches of this function:

for small values of $| \Omega - 4 |$

$$L = a_{-1}^{(i)} (\Omega - 4)^{-1} + a_0^{(i)} + a_1^{(i)} (\Omega - 4) + a_2^{(i)} (\Omega - 4)^2 + \ldots$$
$$(i = 1, 2)$$
$$a_{-1}^{(1)} = - 4 (\beta - 1), \quad a_{-1}^{(2)} = 0, \quad a_0^{(1)} = 3\beta + 1,$$
$$a_0^{(2)} = \beta + 3 [(\beta - 1)^{1/3} + (\beta - 1)^{2/3}]$$
$$a_1^{(1)} = 2, \quad a_1^{(2)} = \frac{1}{4} a_0^{(2)} (\beta - 1)^{1/3} [1 + (\beta - 1)^{1/3} + (\beta - 1)^{2/3}] \times$$
$$[1 + (\beta - 1)^{1/3} + (\beta - 1)^{4/3}]$$
$$a_2^{(1)} = - \frac{1}{16} [48\beta (\beta - 1)^2 + 12 (\beta - 1) (\beta^2 - 9\beta + 9) + \beta^3] (\beta - 1)^{-2}$$

for small values of $| \Omega - 4\beta |$

$$L = a_1^{(3)} (\Omega - 4\beta) + a_2^{(3)} (\Omega - 4\beta)^2 + a_3^{(3)} (\Omega - 4\beta)^3 + \ldots$$
$$a_1^{(3)} = \frac{\beta}{4 (\beta - 1)}, \quad a_2^{(3)} = \frac{\beta - 2}{16 (\beta - 1)^2}, \quad a_3^{(3)} = \frac{8\beta^3 - 9\beta^2 + 3\beta - 1}{64\beta (\beta - 1)^3}$$

for high values of $\Omega > 0$

$$L = a_2^{(i)} \Omega + a_1^{(i)} \Omega^{1/2} + a_0^{(i)} + a_{-1}^{(i)} \Omega^{-1/2} + a_{-2}^{(i)} \Omega^{-1} + a_{-3}^{(i)} \Omega^{-3/2} + \ldots$$
$$(i = 4, 5)$$
$$a_2^{(4)} = a_2^{(5)} = 1, \quad a_1^{(4)} = - a_1^{(5)} = - 2 \sqrt{2 (\beta - 1)}$$
$$a_0^{(4)} = a_0^{(5)} = - \frac{1}{2} (7 - 3\beta)$$

for high values of $| \Omega |$ and $\beta > 9$

$$L = a_{-1}^{(i)} \Omega^{-1} + a_{-2}^{(i)} \Omega^{-2} + a_{-3}^{(i)} \Omega^{-3} + \ldots \quad (i = 6, 7)$$
$$a_{-1}^{(6,7)} = \frac{1}{8} [27 - 18\beta - \beta^2 \pm (\beta - 1)^{1/2} (\beta - 9)^{3/2}] < 0$$

$$a_{-2}^{(6,7)} = \frac{1}{8}\left[-(\beta^3+\beta^2+63\beta-81) \pm (\beta^4-12\beta^3+972\beta+2187)(\beta-1)^{1/2}(\beta-9)^{-1/2}\right]$$

(we will not give the remaining coefficients because the expressions become cumbersome).

The results of our analysis of the sufficient and necessary conditions for stability of the motion (1.2) are shown in Figs.3 and 4. The first (third quadrant of the (L, Ω)-plane

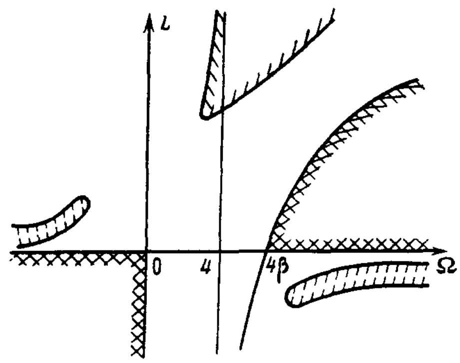

Fig.3

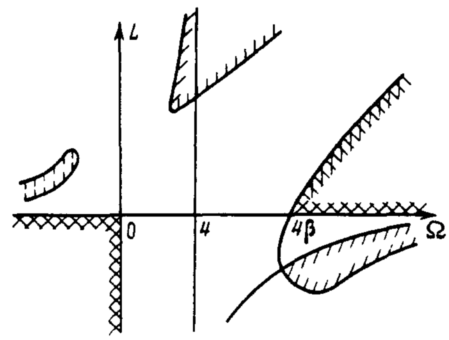

Fig.4

corresponds to motion (1.2) for which the point O is located below point O' and point C above (below) point O; the second (fourth) quadrant corresponds to the motion for which point O is above O', and point C below (above) O. The regions with the boundaries marked by crosses correspond to the motions for which the sufficient and necessary stability conditions are simultaneously satisfied; the regions marked with the inclined dashes correspond to the motions for which only the necessary stability conditions are satisfied. Fig.2 corresponds to $1 < \beta < 9$, and Fig.3 to $9 < \beta < \beta_*$, and at the same time the line marking the stability region which is in the fourth quadrant shifts in the direction opposite to the Ω axis as $\beta \to \beta_*$, and for $\beta = \beta_*$ it becomes tangential to the branch of the curve $L = L(\Omega, \beta_*)$ lying in the first and fourth quadrants. Fig.4 corresponds to values of $\beta > \beta_*$. Parts of the (L, Ω)-plane not marked with dashes refer to the parameters for which the motion (1.2) is unstable, and Eq. (2.2) has at least one pair of complex roots.

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